The Lens Design Using the CMA-ES Algorithm

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Abstract. This paper presents a lens system design algorithm using the covariance matrix adaptation evolution strategy (CMA-ES), which is one of the most powerful self-adaptation mechanisms. The lens design problem is a very difficult optimization problem because the typical search space is a complicated multidimensional space including many local optima, non-linearities, and strongly correlated parameters. There have been several applications of Evolution Algorithms (EA) to lens system designs, and Genetic Algorithms (GAs) are generally expected to be more suitable for these kind of difficult optimization problems than Evolution Strategy(ES) because GAs can provide a global optimization. We demonstrate, however, that a CMA-ES can work better than the GA methods previously applied to lens design problems. Experimental results show that the proposed method can find human-competitive lens systems efficiently.

1 Introduction

The lens system design involves very difficult optimization problems because the search space is typically a complicated multidimensional space including many local optima, non-linearities, and strongly correlated parameters. Modern lens system design is thus generally done with specialized CAD softwares that help designers visualize the lens, evaluate its quality, and locally optimize its variables. Several non-linear optimization methods are available for the local optimization of the lens system (the Newton method, the conjugate gradient method, and the steepest descent method, etc.), and the Damped Least Squares method [3] has been used frequently. But the design processes are highly dependent on the designer's experience in finding the proper starting points because the search space has many local optima. Several global optimization methods, such as the conventional bit-string genetic algorithms, have therefore recently been used in lens system design [3].

The lens system design problems can be formalized as real-valued function optimization problems by fixing the discrete variables such as refractive indexes. Ono et al. applied the real-coded genetic algorithms to lens system design problems [1]. The crossover operator they used was the unimodal normal distribution crossover (UNDX) [5] and their generation-alternation model was the minimal generation gap (MGG) [1] respectively, both of which had shown good performance in benchmark problems for real-valued function optimization.

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There are two kinds of evolutionary algorithms for the optimization of real-valued functions. One is the genetic algorithm (GA), where the solver keeps the multiple search points (a population) and evolves the population so as to cover the promised search area in the search space. The other is the evolutionary strategy (ES), where a single search point is considered and the promised mutation shape is evolved so that it can efficiently generate improved candidates. The GA is generally thought to work better than the ES in the problems that have many local optima because GA can provide global optimization. The ES with covariance matrix adaptation (CMA-ES) [4], which adapts the covariance of mutation shape, has recently been shown to perform well on some benchmark optimization problems which have single peak landscapes.

In this paper we will first apply the CMA-ES to the lens system design problem formalized as real-valued function optimization problems [1] and show that it works better than the real coded-GA. And then we will apply it to the benchmark problems for lens design problem.

Section 2 describes the lens system design problems, explaining both the basis of lens system design and the formalization of the design problems. Section 3 outlines the CMA-ES algorithm. Section 4 gives experimental results and discusses them. We apply the CMA-ES to the benchmark in section 5. Section 6 concludes this paper.

2 The Lens Design Problem

2.1 The Basis of Lens System Design

Given an object of a certain size in the object space, the main task of a lens system is to produce an accurate images of the object. An ideal lens system is defined as follows:

(i) The divergent ray bundles emerging from any single object point on the object plane should be concentrated, after passing through the lens system, to a corresponding image point on the image plane.
(ii) The image of an object on the image plane should be similar to the object on the object plane by a given lateral magnification.

A lens system satisfying these requirements is called an ideal lens system, but an ideal lens system cannot actually be constructed because there are inevitably the differences between a real image and a corresponding ideal image. These differences are aberrations, and the purpose of lens design is to determine the lens arrangement – its surface curvatures, its thickness and its materials – so as to minimize the aberrations.

To calculate the aberrations we need to know real image and corresponding ideal image. The real image is calculated by the procedure called ray trace. Starting at a given point in the object space and proceeding in a given initial direction, a ray trace is the computation of the trajectory of a light ray through the lens system until it reaches the image point on the image plane. The exact ray trace is obtained from the first law of refraction, which governs the behavior of light rays passing through the interface between two media having different refractive indexes. The path of a ray passing from medium 1 to medium 2 obey the following equation:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

where n_1 and n_2 are the refractive indexes of media 1 and 2, and $\theta_1$and θ_2 are incident and refracted angles. By using the first-order approximation of the expansion of a sine function, we can write Eq. (1) as

$$n_1\theta_1 \approx n_2\theta_2 \tag{2}$$

This is the case when rays are close to the optical axis, and this approximation is called the paraxial approximation. When Eq. (2) holds, a lens system having spherical surfaces becomes an ideal lens system. Therefore an ideal image point of an object point is defined as the point at which rays diverging from the object point converge according the paraxial approximation. The calculation of the basic specifications of the lens system − such as a focal length, f-number, and back focal length − can be based on the paraxial approximation.

The cross-section of a lens system having three lens elements is shown in Figure 1. An N-surfaces lens system (N = 6 in Figure 1) can be characterized by the parameters, $c_1, \ldots, c_N, d_1, \ldots, d_N, h_1, \ldots h_N, n_1, \ldots, n_{N+1}$ and s, where c_i is the curvature of the i-th surface and d_i is the distance from the i-th surface to the succeeding one (the N+1-th surface is defined as the image surface). h_i is the aperture of the i-th surface. n_i is the refractive index of the medium occupied between i-th surface and preceding one, and s is the position of the stop. The c_i, s and h_i can take any real value as long as the lens system is physically feasible. Because the refractive indexes are characteristic of materials, however, we can select for the refractive indexes only the values of available materials. Refractive indexes are also wavelength dependent, so three or four refractive indexes corresponding to representative wavelengths in the visible range need to be known.

The f-number, the focal length, and the field size are fundamental specifications that can be explained with reference to Figure 1, which illustrates parallel ray bundles that enter the lens system at field angles of 0 and θ. The line through the centers of curvature of the surfaces is called the optical axis, and the rays passing through the center of the stop are defined as principal rays. Considering the ray bundle parallel to the optical axis, we can see that the focal length is the distance between the focal point C and the vertical position H at which the ray refract. The focal length f is also related to the field angle θ and the image height y. When an object is located infinitely far from the lens system, this relation becomes $y = f\tan\theta$. The f-number, which is related to the brightness of the image, is defined by $F = f/D_{pupil}$, where D_{pupil} is the diameter of the effective ray bundle parallel to the optical axis (i.e. the parallel ray bundle that can reach the image surface). The field size $2w$ is the maximum size of the field that the lens systems should cover. Therefore, we must consider any ray bundle having a field angle θ less than w.

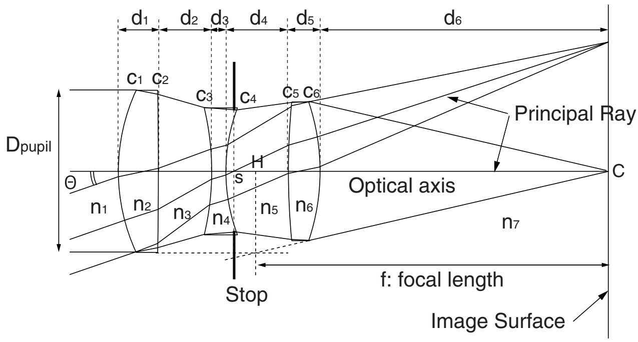

Fig. 1. An example of the lens system.

2.2 Formalization of Lens System Problems

Ono et al.[1] selected the $c_1, \ldots, c_{N-1}, d_1, \ldots, d_{N-1}$ as the control parameters, and characterized the lens system by $2N - 2$ parameters having real values. These parameters were scaled so that the search ranges of curvatures and the distance given in advance were linearly transformed into [0,1]. It should be noted that c_N and d_N were not control variables because they are determined for a lens system to meet a specified focal length. The position of the stop was determined heuristically. Beaulieu et al. [3] further selected the stop position as a control variable. Because their experiments treated monochromatic problems, the refractive indexes were fixed at the helium d wavelength of frequently used glass.

They evaluated the lens systems by using spot diagrams, which are the sets of the points at which the ray bundle diverging from an object point intersect on the image plane. Although as many as possible rays included in the ray bundle should be traced when evaluating the lens system accurately, computational cost are usually reduced by tracing only representative rays including the principal ray and several other several types. Moreover, only three or four representative object points on the object plane are usually selected. When the object is located at infinity, three field angles are selected: $0,\ 0.65w,\ w$. Figure 2 illustrates an example of the spot diagram and the intersection points of the traced rays having the field angle of 0 at the first surface.

To evaluate lens systems in view of the requirements (i) and (ii) for the ideal lens system, Ono et al. defined the resolution R and the distortion D as follows:

$$R = \sum_{\theta \in \{0, 0.65w, w\}} \sqrt{\sum_{k=1}^{M} \{(x_{\theta k} - x_{\theta 0})^2 + (y_{\theta k} - y_{\theta 0})^2\}/M} \tag{3}$$

$$D = \sum_{\theta \in \{0, 0.65w, w\}} \sqrt{(x_{\theta 0} - x_{\theta I})^2 + (y_{\theta 0} - y_{\theta I})^2} \tag{4}$$

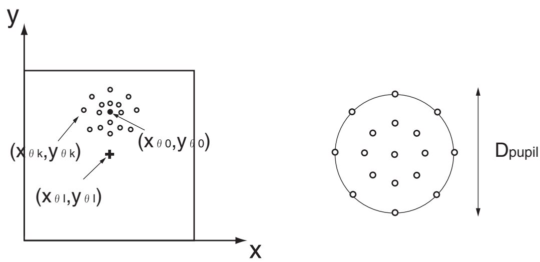

Fig. 2. Spot diagrams and the intersection points of the traced rays at the first surface (the field angle is 0)

where $(x_{\theta I}, y_{\theta I})$ is the ideal image points, $(x_{\theta 0}, y_{\theta 0})$ is the image point of principal ray and $(x_{\theta k}, y_{\theta k})$ are the image points of the other surrounding rays on the image plane, each inclined to the optical axis by an angle θ. In this case $(x_{\theta I}, y_{\theta I})$is defined as $(0, f \tan\theta)$. Given the number of surfaces N, a focal length f, a f-number F, and a field size $2w$, as a specifications, Ono et al. used real-coded genetic algorithms to optimize the lens system under the evaluation function $F = D + R$. They also applied the multi-objective GA using the Pareto optimal selection strategy. Beaulieu et al. [3] applied the bit-string GA and the genetic programming under the similar conditions, where the distortion D was considered as the constraint. In their experiments the distortion had to be less than 1% and the definition of D was different from that specified by Eq. (4). It was

$$D = \max_{\theta \in \{0, 0.65w, w\}} \left\{ \frac{\sqrt{(x_{\theta 0} - x_{\theta I})^2 + (y_{\theta 0} - y_{\theta I})^2}}{\sqrt{(x_{\theta I})^2 + (y_{\theta I})^2}} \right\} \tag{5}$$

3 The CMA-ES Algorithm

The algorithm for the covariance matrix adaptation evolution strategy (CMA-ES) [4], one of the most powerful self adaptation mechanisms available, is the following.

(1) Initialize $g \in R,\ C \in R^{n\times n},\ p_c \in R^n,\ p_\sigma \in R^n,\ \sigma \in R^+$ and $< x >_\mu \in R^n$ as follows:
$g = 0,\ p_c^{(0)} = p_\sigma^{(0)} = 0,\ C^{(0)} = I$ (unity matrix). $\sigma^{(0)}$ and $< x >_\mu^{(0)}$ are given as inputs.

(2) At each generation $g + 1$, compute the λ offspring by using Eq. (6), where z_k are $\mathcal{N}(0, I)$ distributed (elements of z_k are independently $\mathcal{N}(0, 1)$ distributed).

$$x_k^{(g+1)} =< x >_\mu^{(g)} + \sigma^{(g)} B^{(g)} D^{(g)} z_k^{(g+1)}, \qquad k = 1, \ldots, \lambda \tag{6}$$

(3) Select the μ best offspring among them (the set of indices of the selected offspring is denoted as $I_{sel}^{(g)}$ and compute $< x >_\mu^{(g+1)}$ and $< z >_\mu^{(g+1)}$.

$$< x >_\mu^{(g+1)} = 1/\mu \sum_{I_{sel}^{(g+1)}} x_i^{(g+1)} \tag{7}$$

$$< z >_\mu^{(g+1)} = 1/\mu \sum_{I_{sel}^{(g+1)}} z_i^{(g+1)} \tag{8}$$

(4) Update the global step size σ and matrices B and D as follows:

$$p_c^{(g+1)} = (1 - c_c)p_c^{(g)} + \sqrt{c_c(2 - c_c)}\sqrt{\mu}/\sigma^{(g)}(< x >_\mu^{(g+1)} - < x >_\mu^{(g)}) \tag{9}$$

$$C^{(g+1)} = (1 - c_{cov})C^{(g)} + c_{cov} p_c^{(g+1)}(p_c^{(g+1)})^T \tag{10}$$

$$p_\sigma^{(g+1)} = (1 - c_\sigma)p_\sigma^{(g)} + \sqrt{c_\sigma(2 - c_\sigma)}\sqrt{\mu}B^{(g)} < z >_\mu^{(g+1)} \tag{11}$$

$$\sigma^{(g+1)} = \sigma^{(g)} \exp\left(\frac{1}{d_\sigma} \frac{\|P_\sigma^{(g+1)}\| - \hat{\chi_n}}{\hat{\chi_n}}\right) \tag{12}$$

The matrices B and D are determined so as to satisfy

$$C^{(g)} = B^{(g)}D^{(g)}(B^{(g)}D^{(g)})^T \text{ and } C^{(g)}b_i^{(g)} = (d_{ii}^{(g)})^2 b_i^{(g)} \tag{13}$$

(5) Increment g and go to step (1)

The parameter setting is discussed in [4] and the default setting is as follows:

$$c_c = \frac{4}{n+4}, \quad c_{cov} = \frac{2}{(n+\sqrt{2})^2}, \quad c_\sigma = \frac{4}{n+4}, \quad d_\sigma = c_\sigma^{-1} + 1 \tag{14}$$

The term $< x >_\mu^{(g)}$ in Eq. (7) represents the mean vector of the selected individuals of generation g that becomes the center of search point of next generations. The offspring is created by adding a distributed vector $\mathcal{N}(0, \sigma C)$ to $< x >_\mu^{(g)}$, where σ is the global step size and C is the covariance matrix. The term σ and C respectively represent the overall variance and distribution shape of the mutation. These two factors are adapted separately because they should change on different time scales. The covariance matrix C is adapted by Eq. (9) and Eq. (10), and the global step size σ is adapted by Eq. (11) and Eq. (12) respectively. The covariance matrix C is updated with $p_c^{(g+1)}(p_c^{(g+1)})^T$, which is a symmetric matrix of rank one. p_c is the evolution path that is the weighted accumulation of the previously successful mutation. The evolution path often speeded-up the adaptation of the covariance matrix C. The global step size σ is updated with p_r in Eq. (12) where $\hat{\chi_n}$ is expectation of the length of a $(0, I)$-normally distributed random vector which is approximated by $\hat{\chi_n} \approx \sqrt{n}(1 - \frac{1}{4n} + \frac{1}{21n^2})$. The p_σ is also the evolution path calculated by Eq. (11) that is similar to Eq. (9) except that the scaling matrix D is omitted.

The distributed vector $\mathcal{N}(0, C)$ can be created by BDz, where z is the $\mathcal{N}(0, I)$ distributed vector and BD satisfies Eq. (13). The matrix B is the orthogonal $n \times n$ matrix whose i-th columns are normalized eigenvectors of the covariance matrix C, and the matrix D is the $n \times n$-diagonal matrix whose elements d_{ii} are square roots of eigenvalues of C. The main drawback of the CMA algorithm is that the computation time for a single cycle of the algorithm is $O(N^3)$, where N is the dimension of the search space. Several modified algorithm CMA-ES algorithms have therefore been proposed with the aim of reducing the computational time [7,8].

4 Experiments and Results

To evaluate the capability of the CMA-ES for the lens design problems, we apply it to the lens design problems and compare the results with those of representative real-coded GAs.

4.1 Problems

In the experiments we choose two specifications of lens systems used in Ono's experiments [1]. For each specifications we design the lens systems having 4 or 6 lens elements. The specifications are given as follows.

- Specification 1: The focal length, $f = 100mm$, the F number, $F/2.0$, and the field angle, $2w = 45.0\,\text{deg}$. The refractive indexes of all lens are fixed at 1.613.
- Specification 2: The focal length, $f = 100mm$, the F number, $F/3.0$, and the field angle, $2w = 38.0\,\text{deg}$. The refractive indexes of all lens are fixed at 1.613.

The evaluation function is the resolution defined by Eq. (3) and the distortion defined by Eq. (5) is used as a constraint that it must be less than 1%.

4.2 Parameter Setting

N-surfaces lens system are represented by vector $x = (c_1, \ldots, c_{N-1}, d_1, \ldots, d_{N-1}) \in [0, 1]^{2N-2}$. The search range of each parameter is given in advance and linearly transformed into $[1, 0]$. The search ranges of the curvatures c_i and the distances d_i are set to $[-0.66, 0.66]$ (the absolute values of radius are larger than 15.0) and $[0.0, 20.0]$ respectively. The position of stop is determined heuristically in the same way as the Ono's experiments [1].

The CMA-ES uses the parameter setting $\lambda = 20$, $\mu = 4$, and $\sigma^{(0)} = 10.0$. 100 trials are performed for each problem, where each run is stopped if converged.

In Ono's experiments, GA consisted of UNDX and MGG. UNDX-m crossover has been proposed as an generalization of the UNDX and shown a better performance on some benchmark [6]. So we apply UNDX-m, where m is set to 1,2, and 4 (UNDX(m=1) is equal to UNDX). Moreover we try two kind of alternation-generation model, one is the original MGG where the best and the roulette-selection individuals survive to the next generation. and the other is modified MGG where the best two individuals survive. The later has strong selection pressure. Though we applied the 6 type of GAs, only the results of two GAs are shown. One is the GA with UNDX and the original MGG model, that is used in Ono's experiments. The other is the GAs with UNDX-m($m = 4$) and the modified MGG, that showed the best performance among them. 30 trials are performed for each problem, where each run is stopped if converged.

4.3 Results

Figure 3 shows the convergence curve on the 6-elements lens design of the specification 1. The horizontal axis and vertical axis respectively represent the number of evaluations and the resolution of the best individual obtained so far. The CMA-ES can converge about 40 times faster than the GAs. Moreover, the qualities of the lens systems obtained by CMA-ES are not inferior to those of the GAs as shown Figure 4.

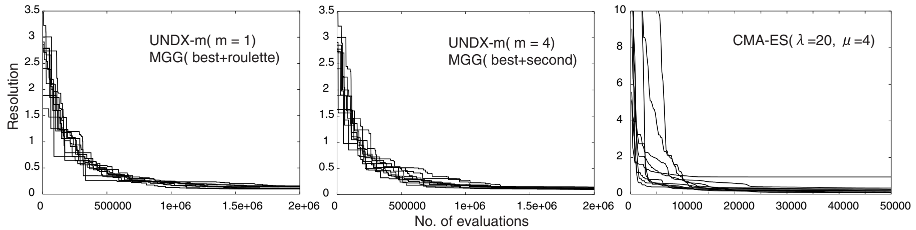

Fig. 3. The converge curve of each optimization method. The 6-elements lens design problems on the specification, $f = 100mm, F/2.0, 2w = 45.0\,\text{deg}$

Fig. 4 shows the qualities of the lens systems obtained by each method. The horizontal axis and vertical axis respectively represent the resolution and the frequency, where the number of trials of the CMA-ES and the GAs are 100 and 30 respectively. Fig. 5 shows the best lens systems obtained by CMA-ES.

The computation time for a single run of the CMA-ES is about 300 sec when applied to the 6-elements lens design of the specification 1. The GAs converge about 40 times slower than the CMS-ES. The algorithms are implemented by C++ language on Xeon processors at 1.7 GHz.

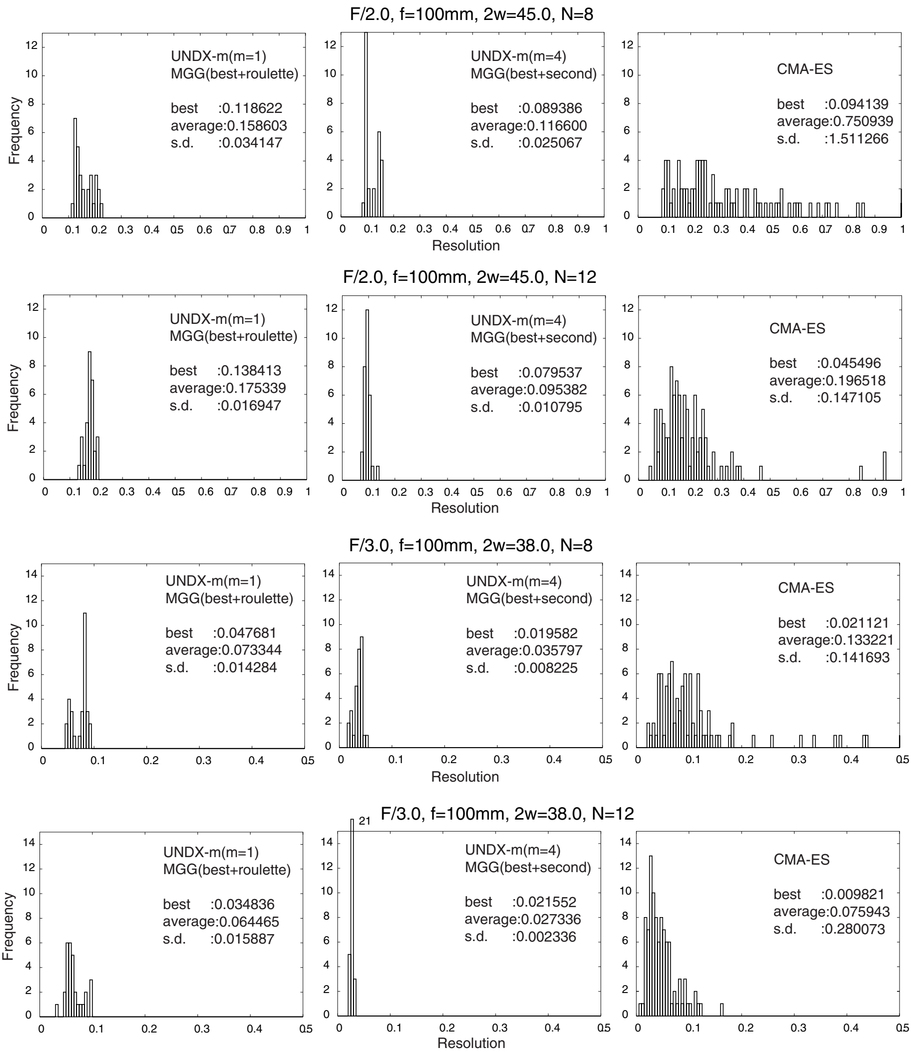

Fig. 4. The frequency of the best solutions. GAs perform 30 trials and CMA-ES performs 100 trials.

4.4 Discussion

The figure 4 shows that the GAs always produce the high quality solutions. Though the averages of resolution obtained by CMA-ES are significantly worse than those of the GAs, the CMA-ES can produce high quality lens systems more efficiently than the GAs because the number of evaluations of CMA-ES requited for the convergence is much smaller than those of the GAs (See Figure 3). The GA with UNDX-m(m = 4) and MGG(best two) work better than GA

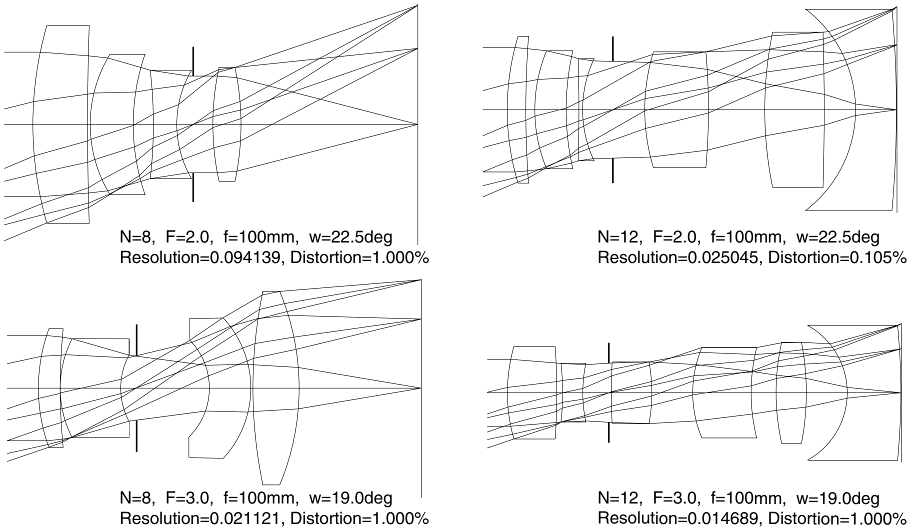

Fig. 5. The best lens design obtained by GAs and CMA-ES

with UNDX and MGG(best+roulette). The results indicate that the GA that converges rapidly tend to work well on the lens system design problems. The landscapes of the lens system are very complicated, therefore crossovers have a few chance to generate improved offspring from parents located remotely in the search space because there is no correlation between such parents. Therefore the population need to converge to a certain degree. On the other hand CMS-ES can adjust the global step size rapidly and locally estimates the landscape. Moreover the results shows that the CMA-ES doesn't easily fall into local optimum

5 Application to the Benchmark

Beaulieu et al.[3] applied the bit-string GA to the benchmark for the lens design problems. The problem is stated in the 1990 International Lens Design Conference (ILDC) [9] and human experts made several solutions, one of these seem to be global optimum. The specification of the lens system is as follows:

– Specification 3: The focal length, $f = 100mm$, the F number, $F/3.0$, and the field angle, $2w = 30.0\,\text{deg}$. The refractive indexes of all lens are fixed at 1.51680. The number of lens elements is 4.

We apply the CMA-ES with the same parameter setting and the evaluation function described in the previous section. The search ranges of the curvatures c_i and the distances d_i are set to $[-0.66, 0.66]$ and $[0.0, 75.0]$ respectively. We

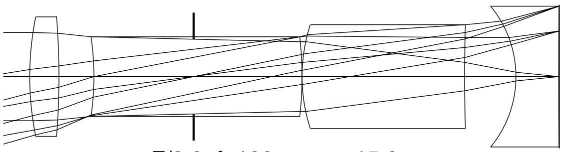

Fig. 6. The best lens design obtained by the CMA-ES under the specification $f = 100mm, F/3.0, 2w = 30.0\,\text{deg}, N = 8$

Table 1. Parameters of the best lens design obtained by the CMA-ES. The stop position is given in the column of thickness.

surface	radius	thickness	aperture	fractive index
1	112.865308	10.887726	22.383670	1.000000
2	-276.900906	12.930491	22.383670	1.516800
3	-103.878518	77.560903	15.018847	1.000000
4	-127.478825	0.005718	15.018847	1.516800
5	63.343159	60.486656	19.410025	1.000000
6	671.192334	19.317317	19.410025	1.516800
7	-41.532186	15.531193	26.488189	1.000000
8	3805.818724	0.198941	26.488189	1.516800
stop		61.012946	14.064979	

performed 300 trials for this problem. The averaged computation time for a single run is 113 seconds on the same processors.

Fig. 6 and Table 1 show the best lens system obtained by 300 trials. The resolution of the best solution is 0.008942. The resolution of the best 1990 ILDC lens system is 0.010434 in our used evaluation function. The best lens system obtained in this experiment is better than the best presented at the 1990 ILDC. Though the evaluation function may be differ a bit from the one used in 1990 ILDC because of the flexibility for selecting tracing rays in the process of the ray trace, we can conclude that the CMA-ES can find the human-competitive solutions efficiently.

6 Conclusion

In this paper we applied the CMA-ES to the lens system design problems and demonstrated that this method can provide high quality solution efficiently better than the representative real-coded GAs. Moreover the human-competitive solution was found on the benchmark of the 4-element lens design problem. GAs are generally expected to be suitable for the problems whose landscape has a lot of local optima And the ES is expected to converge rapidly to the local optimum. Many papers have reported these features on the benchmarks. In the lens system design problem, the CMA-ES and the GAs have almost same capabilities of reaching at good solution even if the landscapes are very complex. More over, the CMA-ES converges much faster than the GAs and doesn't easily fall into local optima. We conclude that the landscape of the lens system design problems are very complicated and then crossovers cannot generate improved offspring from parents located remotely in the search space because there is no correlation between such parents. In this case the ES can perform better than GAs.

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